$$\cos x = \sum_{n=0}^{\infty} c_n x^n \qquad (a=0) \qquad c_n = \frac{f^{(n)}(0)}{n!}$$

$$f(x) = \cos x \Rightarrow f(0) = \cos 0 = 1$$

$$f'(x) = -\operatorname{sen} x \Rightarrow f'(0) = -\operatorname{sen} 0 = 0$$

$$f''(x) = -\cos x \Rightarrow f''(0) = -\cos 0 = -1$$

$$f'''(x) = \operatorname{sen} x \Rightarrow f'''(0) = \operatorname{sen} 0 = 0$$

$$f^{(4)}(x) = \cos x$$

$$\Rightarrow \cos x = 1 + 0 \cdot x - \frac{1}{2!} x^2 + \frac{0}{3!} x^3 + \frac{1}{4!} x^4 + \frac{0}{5!} x^5 - \frac{1}{6!} x^6 + \frac{0}{7!} x^7 + \cdots$$

$$= 1 - \frac{1}{2!} x^2 + \frac{1}{4!} x^4 - \frac{1}{6!} x^6 + \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} x^{2n}$$

$$\left| \frac{x_{n+1}}{x_n} \right| = \left| \frac{(-1)^{n+1} x^{2(n+1)}}{[2(n+1)]!} \cdot \frac{(2n)!}{(-1)^n x^{2n}} \right| = \left| (-1)\, x^2 \cdot \frac{1}{(2n+2)(2n+1)} \right|$$

$$= x^2 \cdot \frac{1}{(2n+2)(2n+1)} \xrightarrow{n \to \infty} 0 < 1$$ $\therefore$ converge p/ todo $x \in \mathbb{R}$.

Analogamente,

$$\operatorname{sen} x = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} x^{2n+1}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} x^{2n} \quad , \quad \operatorname{sen} x = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} x^{2n+1}$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} \cdot x^n$$

$$\cos x = 1 + \frac{0}{1!} x^1 - \frac{1}{2!} x^2 + \frac{0}{3!} x^3 + \frac{1}{4!} x^4 + \frac{0}{5!} x^5 - \frac{1}{6!} x^6 + \frac{0}{7!} x^7 + \dots$$

$$\operatorname{sen} x = 0 + \frac{1}{1!} x^1 + \frac{0}{2!} x^2 - \frac{1}{3!} x^3 + \frac{0}{4!} x^4 + \frac{1}{5!} x^5 + \frac{0}{6!} x^6 - \frac{1}{7!} x^7 + \dots$$

$$e^x = 1 + \frac{1}{1!} x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \frac{1}{4!} x^4 + \frac{1}{5!} x^5 + \frac{1}{6!} x^6 + \frac{1}{7!} x^7 + \dots$$

Fórmula de Euler: $e^{ix} = \cos x + i \operatorname{sen} x$ ✓

$$\cos x = 1 + \frac{0}{1!} x^1 - \frac{1}{2!} x^2 + \frac{0}{3!} x^3 + \frac{1}{4!} x^4 + \frac{0}{5!} x^5 - \frac{1}{6!} x^6 + \frac{0}{7!} x^7 + \dots$$

$$i \operatorname{sen} x = 0i + \frac{i}{1!} x^1 + \frac{0i}{2!} x^2 - \frac{i}{3!} x^3 + \frac{0i}{4!} x^4 + \frac{i}{5!} x^5 + \frac{0i}{6!} x^6 - \frac{i}{7!} x^7 + \dots$$

$$e^{ix} = 1 + \frac{1}{1!} ix + \frac{1}{2!}(ix)^2 + \frac{1}{3!}(ix)^3 + \frac{1}{4!}(ix)^4 + \frac{1}{5!}(ix)^5 + \frac{1}{6!}(ix)^6 + \frac{1}{7!}(ix)^7 + \dots$$

$$= 1 + \frac{i}{1!} x - \frac{1}{2!} x^2 - \frac{i}{3!} x^3 + \frac{1}{4!} x^4 + \frac{i}{5!} x^5 - \frac{1}{6!} x^6 - \frac{i}{7!} x^7 + \dots$$